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DISTANCE AND DISTANCE LAPLACIAN SPECTRUM OF THE ZERO-DIVISOR GRAPH ON THE RING OF INTEGERS MODULO n

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ABSTRACT. For a commutative ring R with non-zero identity, let $Z^*(R)$ denote the set of non-zero zero-divisors of R. The zero-divisor graph of R, denoted by $\Gamma(R)$, is a simple undirected graph with all non-zero zero-divisors as vertices and two distinct vertices $x, y \in Z^*(R)$ are adjacent if and only if $xy = 0$. In this paper, we describe the computation of distance, distance Laplacian spectrum of $\Gamma(\mathbb{Z}_n)$ by exploring its combinatorial structure as the joined union of its induced subgraphs.

1. INTRODUCTION

In this paper G denotes a simple, finite, undirected and connected graph with vertex set $V(G)$ and edge set $E(G)$. The order of a graph G is the cardinality of $V(G)$. If u and v are distinct vertices in a graph G, $d_G(u,v)$ denotes the distance between u and v; which is the length of a shortest path between u and v. Clearly $d_G(u,u) = 0$ and $d_G(u,v) = \infty$ if there is no path between u and v. If $u \in V(G)$, the open neighborhood of u; denoted by $N_G(u)$ is the set of vertices adjacent to u in G. The cardinality of $N_G(u)$ is the degree of u. In a connected graph G, the transmission degree of a vertex v is defined as $Tr(v) = \sum_{u \in V(G)} d_G(u,v)$. The adjacency matrix, $A(G)$ of a graph G of order n is a $0 \quad 1$ matrix of order $n \times n$ with entries a_{ij} such that a_{ij} is 1, if the i th and j th vertices are adjacent, and 0 otherwise.

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