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# Forbidden Subgraph Characterizations of Extensions of Gallai Graph Operator to Signed Graph

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***Abstract.*** We introduce three types of extension of Gallai graph operator in to signed graphs- Gallai signed graph, product-Gallai signed graph and dot-Gallai signed graph. We find the forbidden subgraph characterizations of Gallai signed graph, product-Gallai signed graph and dot-Gallai signed graph.

***Keywords:*** Gallai graph, signed graph

***AMS Mathematics Subject Classification (2010):*** 05C22

## 1. Introduction

A signed graph is obtained from a graph when one regards some of the lines as positive and the remaining lines as negative [7]. Precisely, a signed graph is a pair $(G,\sigma)$ where $G$ is called the underlying graph and $\sigma : E(G) \rightarrow \{+,-\}$ is called the signature function or sign to the edges. The collection of all positive edges and the collection of all negative edges are denoted by $E^+(S)$ and $E^-(S)$, respectively. In social psychology, signed graphs have been used to model social situations (examples in [13], [14] and [9]). A signed graph in which all the edges are positive (negative) is called all-positive (all-negative) signed graph. A signed graph is said to be homogeneous if it is either all-positive or all-negative and heterogeneous, otherwise. A cycle in a signed graph S is said to be positive if the product of the signs of its edges is positive. Otherwise, it is called negative [6]. Similarly, a path in a signed graph is said to be positive, if the product of the signs of the edges is positive and is negative, otherwise. A vertex in a signed graph is considered as a homogeneous vertex if the entire edges incident to it has the same sign. Otherwise, it is a heterogeneous vertex. Further, in [4], every signed graph $S = (G,\sigma)$ can be associated with a signing of its vertices by the function, called the canonical marking of S, defined by the rule,

$$\mu_\sigma(x) = \prod_{E_j \in E_x} \sigma(E_j)$$

where $E_x$ denote the set of all edges of $S$, which are incident on the vertex *'x'.* In literature signed graph in short called as 'sigraph' [10]. In this paper, canonical marking is used to sign the vertices.

The line graph [8] $L(G)$, of a graph $G$ has the edges of $G$ as its vertices and two distinct edges of $G$ are adjacent in $L(G)$ if they are incident in $G$. The Gallai graph $\Gamma(G)$, of a graph $G$ has the edges of $G$ as its vertices and two distinct edges of $G$ are adjacent in $\Gamma(G)$ if they are incident in $G$, but do not span a triangle in $G$ [16]. Though, $\Gamma(G)$ is a spanning subgraph of $L(G)$, their behaviors are different. For example, $L(G)$ has a forbidden subgraph characterization, whereas Gallai graphs do not have the vertex hereditary property and hence cannot be characterized using forbidden subgraphs [16]. But, in [3] it has been proved that there exists a finite family of forbidden subgraphs for the Gallai graphs and the anti-Gallai graphs to be $H$-free for any finite graph $H$. Also it has been proved in [1] that the recognition of anti-Gallai graphs is NP-complete. In [2], the forbidden subgraph characterizations of $G$ for which $\Gamma(G)$ and $\Delta(G)$ is a split graph and is a threshold graph are given.

Signed line graph $L(S)$ of a given signed graph $S = (G,\sigma)$ as defined by Behzad and Chartrand [12] is the signed graph with standard line graph $L(G)$ of $G$ as its underlying graph and whose edges are assigned the signs according to the rule: for any $e_ie_j \in E(L(S))$, $e_ie_j \in E^{-}(L(S))$ if and only if the edges $e_i$ and $e_j$ of $S$ are both negative in $S$. For a signed graph $S$ the set of all signed graphs $S'$ with $L(S') \doteq S$ is called ***signed line roots*** of $S$ [11].

There are two other notions of a signed line graph of a given signed graph $S = (G,\sigma)$ in [5] - product-line sigraph $L(S)$ and dot-line sigraph $L_{\bullet}(S)$. Both of them have $L(G)$ as its underlying graph and only the rule to assign signs to the edges of $L(G)$ are different. In $L_{\times}$, an edge $ee'$ has sign $\sigma(e)\sigma(e')$ and in $L_{\bullet}(S)$, any edge $ee'$ has the sign of the vertex common to $e$ and $e'$ [6]. For a signed graph $S$ the set of all signed graphs $S'$ with $L(S') \doteq S$ is called ***$L_{\bullet}$-roots*** of $S$ [5].

**1.1. New terminology and definitions**

Motivated from the above concepts, we define Gallai signed graph $\Gamma(S)$ of a given signed graph $S = (G,\sigma)$, as the signed graph with the Gallai graph $\Gamma(G)$ as its underlying graph and whose edges are assigned the signs according to the rule: for any $e_ie_j$ in $E(\Gamma(S))$, $e_ie_j$ is negative if and only if the edges $e_i$ and $e_j$ are both negative in $S$ and positive otherwise. Like the concept of signed line roots we introduce the concept of ***Gallai Signed roots*** as the set of all signed graphs $S'$ with $\Gamma(S') \doteq S$ is called ***Gallai Signed roots*** of $S$. In this paper there is no ambiguity to call ***Gallai Signed roots*** as ***roots***.

Similarly, given signed graph $S = (G,\sigma)$, the product-Gallai signed graph $\Gamma_{\times}(S)$ and the dot-Gallai signed graph $\Gamma_{\bullet}(S)$, have $\Gamma(G)$ as their underlying graph and the rule to assign signs to the edges are as follows. In an edge $ee'$ has sign $\sigma(e)\sigma(e')$ and in $\Gamma_{\bullet}(S)$ any edge $ee'$ has the sign of the vertex common to $e$ and $e'$. Like the concept of ***$L_{\bullet}$-roots*** we introduce the concept of ***$\Gamma_{\bullet}$-roots***. For a signed graph S the set of all signed graphs $S'$ with $\Gamma_{\bullet}(S') \doteq S$ is called ***$\Gamma_{\bullet}$-roots*** of $S$ and the set of signed graphs $S'$ with $\Gamma_{\bullet}(S')$ contains $S$ as an induced subgraph is called ***$\Gamma_{\bullet}$-semi roots*** of $S$. Note that, ***$\Gamma_{\bullet}$-semi roots*** consist of signed graphs which contains a ***$\Gamma_{\bullet}$-root*** as an induced subgraph.

If a graph $G$ has a property $P$ implies that $G$ cannot have an induced sub-graph isomorphic to $H$, and then $H$ is called a forbidden subgraph for the property $P$ [15].