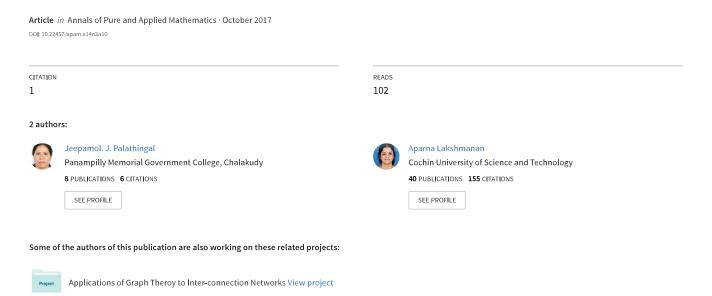
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Forbidden Subgraph Characterizations of Extensions of Gallai Graph Operator to Signed Graph

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Abstract. We introduce three types of extension of Gallai graph operator in to signed graphs- Gallai signed graph, product-Gallai signed graph and dot-Gallai signed graph. We find the forbidden subgraph characterizations of Gallai signed graph, product-Gallai signed graph and dot-Gallai signed graph.

Keywords: Gallai graph, signed graph

AMS Mathematics Subject Classification (2010): 05C22

1. Introduction

A signed graph is obtained from a graph when one regards some of the lines as positive and the remaining lines as negative [7]. Precisely, a signed graph is a pair (G, σ) where G is called the underlying graph and $\sigma: E(G) \to \{+,-\}$ is called the signature function or sign to the edges. The collection of all positive edges and the collection of all negative edges are denoted by $E^+(S)$ and E'(S), respectively. In social psychology, signed graphs have been used to model social situations (examples in [13], [14] and [9]). A signed graph in which all the edges are positive (negative) is called all-positive (all-negative) signed graph. A signed graph is said to be homogeneous if it is either all-positive or allnegative and heterogeneous, otherwise. A cycle in a signed graph S is said to be positive if the product of the signs of its edges is positive. Otherwise, it is called negative [6]. Similarly, a path in a signed graph is said to be positive, if the product of the signs of the edges is positive and is negative, otherwise. A vertex in a signed graph is considered as a homogeneous vertex if the entire edges incident to it has the same sign. Otherwise, it is a heterogeneous vertex. Further, in [4], every signed graph $S = (G, \sigma)$ can be associated with a signing of its vertices by the function, called the canonical marking of S, defined by the rule,

$$\mu_{\sigma}(x) = \prod_{E_j \in E_x} \sigma(E_j)$$

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where E_x denote the set of all edges of S, which are incident on the vertex 'x'. In literature signed graph in short called as 'sigraph' [10]. In this paper, canonical marking is used to sign the vertices.

The line graph [8] L(G), of a graph G has the edges of G as its vertices and two distinct edges of G are adjacent in L(G) if they are incident in G. The Gallai graph $\Gamma(G)$, of a graph G has the edges of G as its vertices and two distinct edges of G are adjacent in $\Gamma(G)$ if they are incident in G, but do not span a triangle in G [16]. Though, $\Gamma(G)$ is a spanning subgraph of L(G), their behaviors are different. For example, L(G) has a forbidden subgraph characterization, whereas Gallai graphs do not have the vertex hereditary property and hence cannot be characterized using forbidden subgraphs [16]. But, in [3] it has been proved that there exists a finite family of forbidden subgraphs for the Gallai graphs and the anti-Gallai graphs to be H-free for any finite graph H. Also it has been proved in [1] that the recognition of anti-Gallai graphs is NP-complete. In [2], the forbidden subgraph characterizations of G for which $\Gamma(G)$ and $\Delta(G)$ is a split graph and is a threshold graph are given.

Signed line graph L(S) of a given signed graph $S = (G, \sigma)$ as defined by Behzad and Chartrand [12] is the signed graph with standard line graph L(G) of G as its underlying graph and whose edges are assigned the signs according to the rule: for any $e_i e_j \in E(L(S))$, $e_i e_j \in E^-(L(S))$ if and only if the edges e_i and e_j of S are both negative in S. For a signed graph S the set of all signed graphs S' with $L(S') \cong S$ is called **signed line roots** of S [11].

There are two other notions of a signed line graph of a given signed graph $S = (G, \sigma)$ in [5] - product-line sigraph L(S) and dot-line sigraph L(S). Both of them have L(G) as its underlying graph and only the rule to assign signs to the edges of L(G) are different. In L_x , an edge ee' has sign $\sigma(e)\sigma(e')$ and in L(S), any edge ee' has the sign of the vertex common to e and e'[6]. For a signed graph S the set of all signed graphs S' with $L(S') \cong S$ is called L-roots of S[5].

1.1. New terminology and definitions

Motivated from the above concepts, we define Gallai signed graph $\Gamma(S)$ of a given signed graph $S = (G, \sigma)$, as the signed graph with the Gallai graph $\Gamma(G)$ as its underlying graph and whose edges are assigned the signs according to the rule: for any $e_i e_j$ in $E(\Gamma(S))$, $e_i e_j$ is negative if and only if the edges e_i and e_j are both negative in S and positive otherwise. Like the concept of signed line roots we introduce the concept of **Gallai Signed roots** as the set of all signed graphs S' with $\Gamma(S') \cong S$ is called **Gallai Signed roots** of S. In this paper there is no ambiguity to call **Gallai Signed roots** as **roots**.

Similarly, given signed graph $S = (G, \sigma)$, the product-Gallai signed graph $\Gamma_{\star}(S)$ and the dot-Gallai signed graph $\Gamma_{\star}(S)$, have $\Gamma(G)$ as their underlying graph and the rule to assign signs to the edges are as follows. In an edge ee' has sign $\sigma(e)\sigma(e')$ and in $\Gamma_{\star}(S)$ any edge ee' has the sign of the vertex common to e and e'. Like the concept of L_{\star} -roots we introduce the concept of Γ_{\star} -roots. For a signed graph S the set of all signed graphs S' with $\Gamma_{\star}(S') \cong S$ is called Γ_{\star} -roots of S and the set of signed graphs S' with $\Gamma_{\star}(S')$ contains S as an induced subgraph is called Γ_{\star} -root as an induced subgraph.

If a graph G has a property P implies that G cannot have an induced sub-graph isomorphic to H, and then H is called a forbidden subgraph for the property P [15].