D 140193 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2026

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half　　　　Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Find the Maclaurian series expansion of $f(z) = \cos z$.

2. Define the term 'removable singularity' of a complex function f.

3. What you mean by the zeros of an analytic function $f(z)$.

4. Does the series $\frac{1+i}{n^2}$ is convergent ? Justify your answer.

5. Find the radius of convergence of the series $\sum_0^\infty (-1)^n z^n/n!$.

6. Evaluate $\int_C \frac{1}{z-1}$ where C is the circle having centre at $1 + i$ and radius $1/2$.

7. Does every continuous functions are differentiable ? Justify your answer.

8. Verify the Cauchy - Riemann equations for the function $f(z) = z^2 - 1$ at the point $z = 1 - i$.

9. Show that $\cosh^2 z - \sinh^2 z = 1$.

10. Write the polar form of the Cauchy - Riemann equation.

Turn over

11. Find the Principal value of $\operatorname{Ln}(1-i)$.

12. Does the function $f(z)=\bar{z}$ is analytic ? Justify your answer.

13. Does the function $f(z)=\sin(1/z)$ has an essential singularity at the orgin ? Justify your answer.

14. Find the residue of the function $f(z)=\dfrac{1}{z\left(z^2+1\right)}$ at the point $z=-i$.

15. Evaluate $\lim\limits_{z\to 0}\dfrac{1-z^3}{1+z+z^2}$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Overall Ceiling is 35.

16. Does the equation $\cosh 2z=2$ has a solution ? If so find the solution.

17. Define an entire function. Does the function $f(z)=\sin z$ is an entire function ? Justify your answer.

18. Prove that $\lim\limits_{n\to\infty} z_n \to 0$ if and only if $\lim\limits_{n\to\infty} |z_n| \to 0$.

19. Show that the function $f(z)=\bar{z}$ continuous on the whole complex plane.

20. Use L'Hospital's rule find

    (i) $\lim\limits_{z\to i}\dfrac{1+z^6}{1+z^{10}}$,

    (ii) $\lim\limits_{z\to 0}\dfrac{1-\cos z}{z^2}$.

21. Suppose that $f(z)$ and $\overline{f(z)}$ are analytic in a domain D. Show that $f(z)$ is constant in D.

22. Find the circle of convergence of each of the following power series :

    (i) $\sum_{k=0}^{\infty} \frac{2^k}{3^k + 4^k} z^k$,

    (ii) $\sum_{k=0}^{\infty} \frac{k!}{k^k} z^k$.

23. Use Cauchy's residue theorem to evaluate $\int_C \frac{1}{z^2 \sin z} dz$ where C is the circle $|z| = 1$ in the counter clockwise direction.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1 - a^2}}, a^2 < 1$.

25. For the function $f(z) = \frac{1}{z(z-1)(z-2)}$, find the Laurent series expansion in

    (i) the domain $|z| < 1$;

    (ii) the domain $1 < |z| < 2$; and

    (iii) the domain $|z| > 2$.

26. Show that $u = x^2 - y^2 - x + 1$ is harmonic. Find a harmonic conjugate v of u such that $u + iv$ is analytic.

27. State and Prove cauchy's Integral formula.

(2 × 10 = 20 marks)