D 140195 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2026

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Solve the differential equation $\left(4+t^2\right)\dfrac{dy}{dt}+2ty=4t.$

2. State existence and uniqueness theorem for first order non linear equations.

3. Verify that $u\left(x, y\right)=\cos x \cosh y$ is a solution of the partial differential equation $u_{xx}+u_{yy}=0.$

4. Show that if $y=\phi\left(t\right)$ is a solution of $y'+p\left(t\right)y=0$, then $y=c\phi\left(t\right)$ is also a solution for any value of the constant c.

5. Find the general solution of $y''+5y'+6y=0.$

6. Find the wronskian of $y_1=e^{2t}$ and $y_2=e^{-3t/2}.$

7. Find the longest interval in which the initial value problem $ty''+3y=t, y\left(1\right)=1, y'\left(1\right)=2$ is certian to have a unique twice differentiable solution.

8. Find the solution of $y''-2y'+y=0.$

9. Find a particular solution of $y''-2y'-3y=3e^{2t}.$

Turn over

10. Evaluate the improper integral $\int_1^{\infty} e^{ct}\, dt.$ For what values of c does this improper integral converge.

11. Find the inverse Laplace transform of $F(s) = \dfrac{2s+2}{s^2+2s+5}$.

12. Prove that convolution integral is commutative.

13. State Fourier convergence theorem.

14. Prove that product of an even function and an odd function is odd.

15. Find the steady state solution of the heat conduction equation $\alpha^2 u_{xx} = u_t$ that satisfies the boundary conditions $u_x(0, t) = 10, u(L, t) = 0.$

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35.

16. Solve $\dfrac{dy}{dx} = \dfrac{x^2 + xy + y^2}{x^2}$.

17. Find an integrating factor and solve the equation $\left(3x^2y + 2xy + y^3\right) + \left(x^2 + y^2\right)y' = 0.$

18. Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0.$ then show that $y = c_1t^2 + c_2t^{-1}$ is also a solution of this equation for any c_1 and c_2.

19. State Abel's theorem and use this theorem to find the wronskian of two solutions of the equation $t^2y'' - t(t+2)y' + (t+2)y = 0.$

20. Find the general solution of $y'' - y' - 2y = -2t + 4t^2$.

21. Use the Laplace transforms to solve the initial value problem $y'' - y' - 6y = 0, y(0) = 1, y'(0) = -1.$

22. Solve the boundary value problem $y'' + 2y = 0,\ y(0) = 0,\ y(\pi) = 0.$

23. Find the Fourier series for $f(x) = \begin{cases} i & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases},\ f(x+4) = f(x).$

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. Let $\phi_0(t) = 0$ and use the method of successive approximations to solve the initial value problem $y' = ty + 1,\ y(0) = 0.$

25. Find a series solution of the equation $y'' + y = 0,\ -\infty < x < \infty.$

26. Find the solution of the initial value problem $y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi),\ y(0) = 0,\ y'(0) = 0.$

27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)