

D 140191

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Name.....

Reg. No.....

**SIXTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
APRIL 2026**

Mathematics

MTS6B10—REAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. State discontinuity criterion for a function at a point.
2. Give an example of a continuous function which is not bounded.
3. Define Lipschitz function.
4. Define norm of a partition. Let $I = [0, 4]$. Calculate the norm of the partition $\mathcal{P} = (0, 1, 2, 4)$.
5. Prove that any step function is integrable.
6. State and prove product theorem.
7. Define null set and give an example of a null set.
8. State first form of Fundamental theorem of calculus.
9. Define uniform convergence of a sequence of functions.
10. Evaluate $\lim_{x \rightarrow \infty} \frac{nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.
11. State Cauchy criterion for uniform convergence of a series of functions.
12. Evaluate $\int_{-1}^0 \frac{dx}{\sqrt[3]{x}}$.

Turn over

13. Define Cauchy principal value of an improper integral.
14. Define integral test.
15. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. State and prove boundedness theorem.
17. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $\epsilon > 0$, prove that there exists a step function $s_\epsilon : I \rightarrow \mathbb{R}$ such that $|f(x) - s_\epsilon(x)| < \epsilon$ for all $x \in I$.
18. State and prove squeeze theorem for Riemann integrals.
19. If $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$, then prove that $f \in \mathcal{R} [a, b]$.
20. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.
21. Prove that all $q \in \mathbb{R}$, $\int_1^\infty x^q e^{-x} dx$ converges.
22. Evaluate $\int_0^\infty \sqrt[3]{ye^{-y^2}} dy$.
23. Prove that $\Gamma(2p) = \frac{2^{2p-1/2}}{\sqrt{2\pi}} \Gamma(p) \Gamma(p+1/2)$.

Section C

Answer any two questions.

Each question carries 10 marks.

Maximum 20 marks.

24. (a) State and prove uniform continuity theorem.
- (b) Prove that $g(x) = \sqrt{x}$ is uniformly continuous on $A = [0, \infty)$.
25. Let $f : [a, b] \rightarrow \mathbb{R}$ and let $c \in (a, b)$. Prove that $f \in \mathcal{R}[a, b]$ if and only if its restriction to $[a, c]$ and $[c, b]$ are both Riemann integrable and in this case $\int_a^b f = \int_a^c f + \int_c^b f$.
26. Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f . Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$.
27. State Cauchy test. Prove that $\int_0^\infty \frac{\sin x}{x} dx$ exists.