D 140194 **(Pages : 3)** **Name.....**

Reg. No.....

SIXTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
APRIL 2026

Mathematics

MTS6B12—CALCULUS OF MULTI VARIABLE

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 25.

1. What is a level curve of a function of two variables ?
2. Let $f(x, y) = x^2 + 3xy - 2x + 3$. Find $f(2, 1)$ and $f(x, y + k)$.
3. Evaluate $\lim_{(x, y, z) \to \left(\frac{\pi}{2}, 0, 1\right)} \frac{e^{2y}(\sin x + \sin y)}{1 + y^2 + z^2}$.
4. Show that the function $u(x, y) - e^x \cos y$ is harmonic in the xy plane.
5. Find the directional derivative of $f(x, y) = 4 - 2x^2 - y^2$ at the point (1, 1) in the direction of the unit vector u that makes an angle of $\frac{\pi}{3}$ radians with the positive x-axis.
6. Find the critical points of $f(x, y) = 3 - \sqrt{x^2 + y^2}$.
7. State Lagrange's theorem.
8. Find an approxiamtion for $\iint_R (x - 4y)\, dA$, where $R = \{(x, y) : 0 \le x \le 2, 0 \le y \le 1\}$, using Reimann sum of $f(x, y) = x - 4y$ over R with $m = n = 2$ and taking evaluation point $\left(x_{ij}^*, y_{ij}^*\right)$ to be the center of R_{ij}.

Turn over

9. Evaluate $\int_0^1 \int_0^2 3x^2 y \, dydx$.

10. State Fubini's theorem for rectangular regions.

11. Find the moments of inertia with respect to the x-axis, the y axis and the origin of a homogeneous disk of mass m and radius a, centered at the origin.

12. Define divergence of a vector field.Find the divergence of F $(x, y) = xi + yj$.

13. Find curl of F $(x, y, z) = xi + yj$.

14. Let F = Pi + Qj + Rk be a vector field in space, and suppose that P, Q and R have continuous second order partial derivatives. Show that *div curl* F = 0.

15. State Green's theorem.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 35.

16. A storage tank has the shape of a right circular cylinder. Suppose that the radius and height of the tank are measured at 1.5 ft and 5 ft, respectively, with a possible error of 0.05 ft and 0.1 ft, respectively. Use differentials to estimate the maximum error in calculating the capacity of the tank.

17. Let W = $x^2y - xy^3$, where $x = \cos t$ and $y = e^t$. Find $\frac{dw}{dt}$ and its value at $t = 0$.

18. Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

19. Find the volume of the solid S lying under the graph of the surface $z = x^3 + 4y$ and above the region R in the xy plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

20. Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

21. Let f be a scalar function, and let **F** be a vector field. If f and the components of F have first order partial derivatives, show that $div\,(f\,\mathbf{F}) = f\ div\ \mathbf{F} + \mathbf{F} \cdot \nabla f$.

22. Find the work done by the force field $F(x, y, z) = -yi + xj + zk$ in moving a particle along the helix C described by the parametric equations $x = \cos t$, $y = \sin t$ and $z = t$ from (1, 0, 0) to $\left(0, 1\,\frac{\pi}{2}\right)$.

23. Evaluate $\oint_C x^2\,dx + (xy + y^2)\,dy$, where C is the boundary of the region R bounded by the graphs of $y = x$ and $y = x^2$ and is oriented in a positive direction.

Section C

Answer any **two** *questions.*

Each question carries 10 *marks.*

Maximum 20 *marks.*

24. Let $w = f(x, y)$, where f has continuous second order partial derivatives and let $x = r^2 + s^2$ and $y - 2rs$. Find $\frac{\partial^2 w}{\partial r^2}$.

25. Find the absolute extreme values of $f(x, y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 \leq 4$.

26. Evaluate $\iiint_T \sqrt{x^2 + z^2}\ dv$, where T is the region bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

27. State Stoke's theorem. Verify Stoke's theorem for the case in which $F(x, y, z) = 3zi + 2xj + y^2k$, S is the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, and C is the trace of S on the xy plane.

(2 × 10 = 20 marks)