

Q.P Code : D143772	Total Pages:2	Name	737143
		Register No.	
<b>FOURTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2026</b>			
<b>MATHEMATICS</b>			
<b>MAT4CJ203 Real Analysis I</b>			
<b>2024 Admission Onwards</b>			
<b>Maximum Time :2 Hours</b>		<b>Maximum Marks :70</b>	

<b>Section A</b>	
<b>All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)</b>	
1	Show that the set of all integers $\mathbb{Z}$ is countable.
2	Define a) A Finite set b) Denumerable Set
3	Show that if $a \in \mathbb{R}$ , then $a \cdot 0 = 0$
4	Find all $x \in \mathbb{R}$ that satisfy $ 4x - 5  \leq 13$
5	Prove that: A sequence in $\mathbb{R}$ can have at most one limit
6	Give an example of two divergent sequences $X$ and $Y$ such that: a) their sum $X + Y$ converges, b) their product $XY$ converges.
7	Give an example of an unbounded sequence that has a convergent subsequence.
8	Write a short note on Properly Divergent Sequences
9	Define Euler's Number
10	State Sequential Criterion for Limits

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## Section B

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All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	<p>Suppose that <math>S</math> and <math>T</math> are sets and that <math>T \subseteq S</math>. Prove that</p> <p>(a) If <math>S</math> is a finite set, then <math>T</math> is a finite set.</p> <p>(b) If <math>T</math> is an infinite set, then <math>S</math> is an infinite set.</p>
12	State and prove Cantor's Theorem
13	Prove that "There does not exist a rational number $r$ such that $r^2 = 2$ "
14	Find all $x \in \mathbb{R}$ that satisfy the equation $ x + 1  +  x - 2  = 7$
15	<p>Let <math>X = (x_n; n \in \mathbb{N})</math> be a sequence of real numbers and let <math>m \in \mathbb{N}</math>. Then prove that the <math>m</math>-tail <math>X_m = (x_{n+m}; n \in \mathbb{N})</math> of <math>X</math> converges if and only if <math>X</math> converges. In this case,</p> $\lim X = \lim X_m$
16	Construct a sequence $(s_n)$ of real numbers that converges to $\sqrt{5}$ and correct to within 4 decimals.
17	State and prove Monotone Subsequence Theorem
18	If $x_1 = 2$ and $x_{n+1} := 2 + \frac{1}{x_n}$ for all $n \in \mathbb{N}$ show that $(x_n)$ is a contractive sequence. Find the limit.

## Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	<p>Prove that the following statements are equivalent:</p> <ol style="list-style-type: none"> <li>1. <math>S</math> is a countable set.</li> <li>2. There exists a surjection of <math>\mathbb{N}</math> onto <math>S</math>.</li> <li>3. There exists an injection of <math>S</math> into <math>\mathbb{N}</math>.</li> </ol>
20	State and prove the Density Theorem of rational numbers

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