D 140689 **(Pages : 2)** **Name..................................**

Reg. No...............................

SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Let X = {1, 2, 3, 4} and $\tau = \{X, \phi, \{1, 2\}, \{1, 2, 3\}\}$. Verify whether τ is a topology on X.
2. Consider $\mathbb{R}$ with cofinite topology. Find the closure of the set $\mathbb{N}$ of all natural numbers in this space.
3. Let X, Y be topological spaces and X × Y be the product space. Let B be open in Y. Verify whether X × B is open in X × Y.
4. Verify whether $\mathbb{R}$ with usual topology is separable.
5. Verify whether $\mathbb{R}$ with discrete topology is connected.
6. Verify whether $\mathbb{R}$ with usual topology is path connected.
7. Give a topology on the reals $\mathbb{R}$ so that it is not a T_2-space.
8. Let $\mathbb{R}$ be the real line and A = {1, 2, 3} and B = {4, 5}. Find a continuous function $f : \mathbb{R} \to [0, 1]$ such that $f(x) = 0$ for all $x \in$ A and $f(x) = 1$ for all $x \in$ B.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage of 2.

Module I

9. Show that if X is a second countable space then every subspace of X is also second countable.
10. Let $f : X \to Y$ be a continuous function. Show that $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq X$.
11. Let X, Y be topological spaces and $f : X \to Y$ be a bijection such that f and f^{-1} are open. Show that f is a homeomorphism.

Turn over

Module II

12. Let $f : X \to Y$ be a continuous surjective map. Show that f is a quotient map.
13. Show that continuous image of a connected space is connected.
14. Show that if A is a connected subset of topological space X then $\overline{A}$ is also connected.

Module III

15. Show that in a Hausdorff space limits are unique.
16. Show that every metric space is a normal space.
17. Give an example of a T_1-space which is not a T_2-space.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. (a) Define subbase of a topology.

    (b) Show that $\mathcal{S} = \{(-\infty, a), (b, \infty) : a, b \in \mathbb{R}\}$ is a subbase for the usual topology of $\mathbb{R}$.

    (c) Let $\mathcal{S}$ be a family of subsets of a set X and τ be the smallest topology on X containing $\mathcal{S}$. Show that $\mathcal{S}$ is a subbase for τ.

19. (a) Define interior of a set in a topological space.

    (b) Let X be a topological space and $A \subseteq X$. Show that $\text{int}(A) = \cup\{G \subseteq A : G \text{ is open}\}$.

    (c) Find $\text{int}(\mathbb{Q})$ in the usual topology of $\mathbb{R}$ where $\mathbb{Q}$ is the set of all rationals.

20. (a) Define compact space.

    (b) Show that every continuous image of a compact space is compact.

    (c) Show that every real valued function on compact space is bounded.

21. (a) Define Hausdorff space.

    (b) Show that every Hausdorff space is a T_1-space.

    (c) Show that every compact Hausdorff space is a T_4-space.

(2 × 5 = 10 weightage)

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