

D 140690

(Pages : 3)

Name.....

Reg. No.....

**SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE
EXAMINATION, APRIL 2026**

Mathematics

MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A (Short Answer Type Questions)*Answer all questions.**Each question carries a weightage of 1.*

- Find the series solution of the differential equation $y' = 2xy$.
- Is $x = 0$ an ordinary point for the differential equation $(1 + x^2)y'' + 2xy' - 2y = 0$.
- Find a regular singular point of the following differential equation on the x -axis :
$$x^3(x-1)y'' - 2(x-1)y' + 3xy = 0.$$
- Prove that $\Gamma(n+1) = n!$ for any integer $n \geq 0$.
- Describe the autonomous system with the help of an example.
- Show that $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ is a solution of the homogeneous system $\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y. \end{cases}$
- Find the exact solution of the initial value problem $y' = y^2, y(0) = 1$.
- Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on the rectangle $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

(8 × 1 = 8 weightage)

Part B (Paragraph Type Questions)*Answer any two questions from each module.**Each question carries a weightage of 2.*

Module I

- Solve the differential equation $x^2y'' + (2-x)y' = 0$.
- Prove that $\lim_{b \rightarrow \infty} F(a, b, a, \frac{x}{b}) = e^x$.

11. Prove that $P_{2n}(0) = (-1)^n \frac{1.3 \dots (2n-1)}{2^n n!}$.

Module II

12. Prove that $\frac{d}{dx} [x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x)$.

13. Find the general solution of $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$.

14. Find a Liapunov function for system $\begin{cases} \frac{dx}{dt} = -2x + xy^3 \\ \frac{dy}{dt} = -x^2 y^2 - y^3 \end{cases}$.

Module III

15. Using Picard's method of successive approximation find first three successive approximate solutions of the initial value problem $y' = x + y$, $y(0) = 0$.
16. Find the shortest curve joining (1, 1) and (3, 4).
17. Let $y(x)$ be a nontrivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a, b]$. Prove that $y(x)$ has at most finite number of zeros in this interval.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Each question carries a weightage of 5.

18. Prove that the differential equation $4x^2 y'' - 8x^2 y' + (4x^2 + 1)y = 0$ has only one Frobenius series solution. Also find the general solution.
19. Find the general solution of the differential equation $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$ near the singular point $x = -1$.
20. Prove that $\int_0^1 x J_p(\lambda_m x) J_p(\lambda_n x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} J_{p+1}(\lambda_n)^2 & \text{if } m = n, \end{cases}$ where λ_m and λ_n are zeros of the Bessel function $J_p(x)$.

21. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| \leq K|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y)$ $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$.

(2 × 5 = 10 weightage)