

D 140691**(Pages : 3)****Name.....****Reg. No.....**

**SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE
EXAMINATION, APRIL 2026**

Mathematics

MTH 2C 10—OPERATIONS RESEARCH

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all questions.

Each question carries a weightage of 1.

1. Show that the sum of two convex functions is a convex function.
2. Determine the maximum number of possible basic solutions to the problem :

Maximize $4x_1 + 3x_2$

subject to $3x_1 + 2x_2 \leq 10$

$5x_1 + x_2 \leq 9$

$x_1 \geq 0, x_2 \geq 0.$

3. What is meant by degeneracy in a linear programming problem ?
4. Prove that the dual of the dual is the primal.
5. Briefly describe about the transportation matrix associated with a transportation problem.
6. What is meant by a mixed integer linear programming problem ?
7. Describe the effect of introducing new constraint on the optimal solution of an LP problem.
8. Briefly describe a zero-sum two-person game.

(8 × 1 = 8 weightage)

Part B

*Answer any **six** questions by choosing **two** questions from each unit.*

Each question carries a weightage of 2.

Unit I

9. Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Prove that $f(X)$ is a convex function if and only if $f(X_2) - f(X_1) \geq (X_2 - X_1)' \nabla f(X_1)$ for all X_1, X_2 in K .

Turn over

10. Show graphically that the problem:

$$\begin{aligned} &\text{Maximize } 3x_1 + 4x_2 \\ &\text{subject to } 4x_1 + 3x_2 \geq 12 \\ &\quad \quad \quad x_1 + 2x_2 \leq 2 \\ &\quad \quad \quad x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

has no feasible extreme points.

11. What is meant by simplex multipliers ? Explain with a suitable example.

Unit II

12. If the primal problem is feasible, then prove that it has an unbounded optimum if and only if the dual has no feasible solution and vice versa.
13. There are forest areas F_1, F_2, F_3, F_4 and timber depots D_1, D_2, D_3 . The following table gives the produce of each forest area, the minimum timber required at each depot to attract buyers, and the cost of transportation per unit of timber from each forest area to each depot. Find the distribution of the entire forest produce for minimum cost of transportation.

	D_1	D_2	D_3	
F_1	3	4	6	100
F_2	7	3	8	80
F_3	6	4	5	90
F_4	7	5	2	120
	110	110	60	

14. Briefly describe the Caterer problem.

Unit III

15. Tasks A, B, ... H. I constitute a project. The notation $X < Y$ means that the task X must be finished before Y can begin. With this notation

$$A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I.$$

Find the minimum time of completion of the project.

16. Briefly describe about parametric linear programming.
17. Solve the game with the payoff matrix

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

(6 × 2 = 12 weightage)

Part C

*Answer any two questions.
Each question carries a weightage of 5.*

18. (a) Let $K \subseteq E_n$ be a convex set $X \in K$ and $f(X)$ a convex function. If $f(X)$ has a relative minimum, then prove that it is also a global minimum.
- (b) Solve the following problem using Simplex method :

$$\begin{aligned} &\text{Maximize } 5x_1 + 3x_2 + x_3 \\ &\text{subject to } 2x_1 + x_2 + x_3 = 3 \\ &\quad -x_1 + 2x_3 = 4 \\ &\quad x_1, x_2, x_3 \geq 0. \end{aligned}$$

19. (a) Briefly describe the applications of duality in linear programming.
- (b) Prove that the transportation problem has a triangular basis.
20. (a) Define the spanning tree of a connected graph.
- (b) Prove that for an $m \times n$ matrix game, both $\max_X \min_Y E(X, Y)$ and $\min_Y \max_X E(X, Y)$ exist and are equal.
21. Solve the following problem by cutting plane method :

$$\begin{aligned} &\text{Minimize } -2x_1 - 3x_2 \\ &\text{subject to } 2x_1 + 2x_2 \leq 7 \\ &\quad 0 \leq x_1 \leq 2 \\ &\quad 0 \leq x_2 \leq 2 \\ &\quad x_1, x_2 \text{ integers.} \end{aligned}$$

(2 × 5 = 10 weightage)