D 143207 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH 4E 09—DIFFERENTIAL GEOMETRY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Sketch the level set $f^{-1}(c)$ at $c=0$ for the function $f(x_1, x_2) = x_1^2 - x_2^2$.

2. Define parametrized curve in $\mathbb{R}^{n+1}$.

3. Find the spherical image of the cylinder $x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$.

4. For the parametrized curve α given by $\alpha(t) = (\cos t, \sin t)$ find the speed $\|\dot{\alpha}\|$.

5. Show that for $a, b, c, d \in \mathbb{R}$ the parametrized curve

$$\alpha(t) = (\cos(at+b), \sin(at+b), ct+d)$$

is a geodesic on the cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$.

6. Let $\bar{X}$ be a Euclidean parallel vector field along a parametrized curve α. Show that $\bar{X} = 0$.

7. Define principal curvature of an oriented n-surface.

Turn over

8. Let $\psi : \mathrm{U} \to \mathbb{R}^3$ be defined by

$$\psi(\theta, \phi) = (r \cos\theta \sin\phi, r \sin\theta \sin\phi, r \cos\phi)$$

where $\mathrm{U} = \{(\theta, \phi) : 0 < \phi < \pi\}$. Find $\mathrm{Im}\psi$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit*
Each question has weightage 2.

Unit I

9. Sketch the vector field $\bar{\mathrm{X}}(p) = (p, \mathrm{X}(p))$ where $\mathrm{X}(p) = p$.

10. Show that $\nabla f(\alpha(t_0)) \cdot \dot{\alpha}(t_0) = 0$ where α is a parametrized curve whose image is contained in $f^{-1}(c)$ and $\alpha(t_0) \in f^{-1}(c)$.

11. Show that the n-plane $a_1x_1 + a_2x_2 + ... + a_{n+1}x_{n+1} = 1$ where $(a_1, a_2, ..., a_{n+1}) \neq 0$ is an n-surface in $\mathbb{R}^{n+1}$.

Unit II

12. Let $\bar{\mathrm{X}}, \bar{\mathrm{Y}}$ be smooth vector fields along a parametrized curve . Show that $\left(\bar{\mathrm{X}} \cdot \bar{\mathrm{Y}}\right) = \bar{\mathrm{X}}\ \bar{\mathrm{Y}} + \bar{\mathrm{X}} \cdot \bar{\mathrm{Y}}$.

13. Show that $\left(\bar{\mathrm{X}} + \bar{\mathrm{Y}}\right)' = \bar{\mathrm{X}}' + \bar{\mathrm{Y}}'$. where $'$ denotes covariant derivative.

14. Let C be a connected oriented plane curve and $\beta : \mathrm{I} \to \mathrm{C}$ be a unit speed parametrization of C. Show that if β periodic then C is compact.

Unit III

15. Show that a parametrized 1-surface is a regular parametrized curve.

16. Let $\psi : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$\psi(\theta, \phi) = ((a + b\cos\phi)\cos\theta, (a + b\cos\phi)\sin\theta, b\sin\phi).$$

For $p = (\theta, \phi)$ find $E_1(p)$ and $L_p(E_1(p))$.

17. Define local parametrization and chart associated with it for an n-surface S.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Define integral curve of a vector field.

(b) Show that every smooth vector field has an integral curve.

19. (a) Define maximal integral curve of a tangent vector field on an n-surface S.

(b) Let S be an n-surface in $\mathbb{R}^{n+1}$ and $\bar{X}$ be a smooth tangent vector field on S. Show that for each $p \in S$ there exists a maximal integral curve on $\bar{X}$ through p.

20. (a) Define Weingarten map L_p of an n-surface S.

(b) Show that $L_p(v) \cdot w = v \cdot L_p(w)$ for all $v, w \in S_p$.

21. (a) Define Gauss-Kronecker curvature $K(p)$ of an n-surface S.

(b) Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$. Show that $K(p) \neq 0$ for all $p \in S$ if and only if the second fundamental form at p is definite for all $p \in S$.

(2 × 5 = 10 weightage)