D 140687 **(Pages : 3)** **Name..............................**

Reg. No..............................

SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH2C06—ALGEBRA—II

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Verify whether $6\mathbb{Z}$ is a maximal ideal of the ring $\mathbb{Z}$ of integers.
2. Find the characteristic of the field $\mathbb{Z}_5$ of the integers mod 5.
3. Verify whether $\sqrt{\pi}$ is algebraic over the field $\mathbb{Q}(\pi)$.
4. Find all automorphisms of the field $\mathbb{Q}\left(\sqrt{2}\right)$.
5. Let σ be the automorphism of the field $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$ such that $\sigma\left(\sqrt{2}\right)=\sqrt{2}$ and $\sigma\left(\sqrt{3}\right)=-\sqrt{3}$. Find the fixed field of σ.
6. Find the splitting field of the polynomial $x^3 - 2$ over $\mathbb{Q}$.
7. Let $K=\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$. Find the subgroup of $G(K/\mathbb{Q})$ that correspond to $E=\mathbb{Q}\left(\sqrt{3}\right)$ in the Galois correspondence.
8. Find the splitting field of the polynomial $x^5 - 1$ over $\mathbb{Q}$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Let F be a field of characteristic zero. Show that F contains a subfield isomorphic to the field $\mathbb{Q}$ of rationals.

Turn over

10. Let $p(x)$ be an irreducible polynomial in $F[x]$ where f is a field. Show that the ideal generated by $p(x)$ is a maximal ideal in $F[x]$.

11. Find the algebraic closure of the rationals $\mathbb{Q}$ in the field $\mathbb{C}$ of complex numbers.

Module II

12. Let E, E′ be field of order p^n where p is a prime and n is a positive integer. Show that E and E′ are isomorphic fields.

13. Let $F \leq F \leq \overline{F}$ be fields where $\overline{F}$ is the algebraic closure of F. Let E be a splitting field over F. Show that if σ is an automorphism of $\overline{F}$ leaving F fixed then $\sigma(a) \in E$ for all $a \in E$.

14. Let p be a prime and $E = \mathbb{Z}_p(y)$ where y is an indeterminate. Let $t = y^p$ *and* $F = \mathbb{Z}_p(t) \leq E$. Show that E is an algebraic extension of F.

Module III

15. Describe all elements of the Galois group $G(K/\mathbb{Q})$ where $K = \mathbb{Q}\left(\sqrt{2}, \sqrt{3}\right)$.

16. Show that the 8th cyclotomic polynomial $\phi_8(x)$ is $x^4 + 1$.

17. Show that regular 18-gon is not cosntructible by ruler and compass.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

18. (a) Define maximal ideal in a ring.

    (b) Let R be a commutative ring with unity and M be an ideal of R. Show that M is a maximal ideal of R if and only if R/M is a field.

19. (a) Show that the set of all constructible real numbers forms a subfield of the field of reals.

    (b) Show that doubling the cube is an impossible geometric construction.

20. Let F be a finite field of characteristic p. Let $\sigma_p : F \to F$ be defined by

$$\sigma_p(a) = a^p \text{ for all } a \in F.$$

Show that :

(a) σ_p is an automorphism of F.

(b) The fixed field of σ_p is isomorphic to $\mathbb{Z}_p$.

21. (a) Define normal extension of a field.

    (b) Let K be a finite normal extension of a field F and E be an extension of F such that $E \leq K$. Show that :

    (i) K is a finite normal extension of E.

    (ii) G(K/E) is a subgroup of G(K/F).

(2 × 5 = 10 weightage)