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D 143209 **(Pages : 3)** **Name....................................**

Reg. No................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH4E11—GRAPH THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *the questions.*

Each question carries a weightage 1.

1. Distinguish between a trail and path. Illustrate with an example.

2. Define block and bridge of a simple graph. Illustrate with an example.

3 Detine a tree. Prove that a (p, q) graph G is a tree if and only if $p = q + 1$.

4. Define diameter of a connected graph. Illustrate with an example.

5. Define chromatic number χ (G) of a graph G. Prove that $\chi(K_n) = n$.

6. Prove that in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.

7. Define line independence number $\beta_1(G)$ of a graph G. Prove that $\beta_1(K_p) = \left[\frac{p}{2}\right]$.

8. Prove that in a k-critical graph $(k \geq 2)$, no vertex cut is a clique.

(8 × 1 = 8 weightage)

Turn over

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Part B

Answer any **two** *questions from each module.*

Each question carries a weightage 2.

Module I

9. Prove that in a tree any *two* vertices are connected by a path.

10. Prove that a graph G is a cut edge of G if and only if e contained no cycle of G.

11. Prove that the number of spanning trees of a complete graph K_n is n^{n-2}.

Module II

12. Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

13. Prove that for any graph G of order n, $\alpha + \beta = n$, where α is the independence number and β is the covering number of G.

14. Prove that every 3-regular graph without cut edges has a perfect matching.

Module III

15. If the graph G is k-critical, then prove that $\delta \geq k - 1$.

16. Prove that K_5 is non-planar.

17. State and prove Euler formula for a is a connected plane graph G.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*

Each question carries a weight of 5.

18. Prove that a non-empty connected graph is Eulerian if and only if it has no vertex of odd degree.

19. Explain the Chinese postman problem.

20. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge coloring in which both colors are represented at each vertex of degree at least two.

21. Prove that every planar graph is 5-vertex colourable.

(2 × 5 = 10 weightage)